

Math 8

Proportions & Similar Triangles

~Study Guide~

- 1) A ratio can be written three ways:

3 to 5

3:5

$\frac{3}{5}$

- 2) A ratio can be simplified just like a fraction:

35 to 15

*What number goes into 35 and 15? (5)

*Divide both numbers in the ratio by 5.

The simplified ratio is 7 to 3.

- 3) Solving Proportions

*To solve a proportion you must cross multiply.

examples:

1) $\frac{x}{10} = \frac{3}{7}$

$$\frac{7x}{7} = \frac{30}{7}$$

$x = 4.3$

2) $\frac{3}{(x+2)} = \frac{2}{(x+5)}$

$$3(x+5) = 2(x+2)$$
$$3x+15 = 2x+4$$
$$\begin{array}{r} 3x+15 = 2x+4 \\ -2x \quad -2x \\ \hline 1x+15 = 4 \\ -15 \quad -15 \\ \hline x = -9 \end{array}$$

Ratio Example:

The ratio of students to parents at a play was ~~5:6~~. There were 150 parents at the play. How many students were at the play?

$$\frac{\text{students}}{\text{parents}} = \frac{5}{6} = \frac{x}{150} \quad \begin{matrix} S \\ P \end{matrix}$$

$$\frac{6x}{6} = \frac{750}{6}$$

$$x = 125 \text{ Students}$$

4) Using proportions to solve problems:

*To set up a proportion, you must label the proportion with words before you label it with numbers.

Example:

Mary averages 15 point per game. She has already played 5 games this season. How many points should Mary score during an 18 game season?

$$\begin{array}{c} \text{pts.} \\ \text{games} \end{array} \frac{15}{1} = \frac{x}{18} \begin{array}{c} \text{pts.} \\ \text{games} \end{array}$$

$$1x = 270$$

$$x = 270$$

Mary should score 270 points during an 18 game season.

5) Using proportions with recipes:

Example:

Lucy is going to bake cookies for the bake sale. The 8th graders are looking to raise money for their D.C. trip. Lucy needs to make 80 cookies. Her recipe for 48 cookies calls for 3 cups of flour. How much flour does Lucy need in order to make 80 cookies?

$$\begin{array}{c} \text{cookies} \\ \text{flour} \end{array} \frac{48}{3} = \frac{80}{x} \begin{array}{c} \text{cookies} \\ \text{flour} \end{array}$$

$$\frac{48x}{48} = \frac{240}{48}$$

$$x = 5 \text{ cups of flour}$$

Lucy needs 5 cups of flour to bake 80 cookies.

There are three ways to solve a problem with proportional relationships:

1. proportion
2. scale factor/table
3. equation

Example:

The ratio of the numbers of A, B, and C exercises in one section of a textbook is 6:3:1. If there are 40 exercises in all, how many are B exercises?

Proportion:
$$\frac{B}{Total} = \frac{3}{10} = \frac{x}{40}$$

$$\frac{10x}{10} = \frac{120}{10}$$

$$x = 12$$

Scale Factor/Table:

A	B	C	Total
6	3	1	10
24	12	4	40

Scale Factor

$$40 \div 10 = 4$$

Equation:

Let x = scale factor

$$6x + 3x + 1x = 40$$

B exercises

$$3x$$

$$3 \cdot 4$$

$$12$$

$$\frac{10x}{10} = \frac{40}{10}$$

$$x = 4$$

∴ There are 12 B exercises.

SIMILAR TRIANGLES

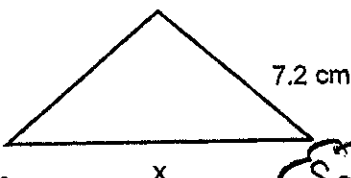
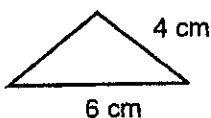
Finding missing sides in similar triangles

Cross-Multiply!

* Use a proportion or scale factor to solve. *

Ex: Find the missing measures in the following similar triangles:

a)



Proportion

$$\begin{array}{ccc} \text{Big} & 7.2 & x \\ \text{Small} & 4 & 6 \end{array}$$

Scale factor:
 $7.2 \div 4 = 1.8$

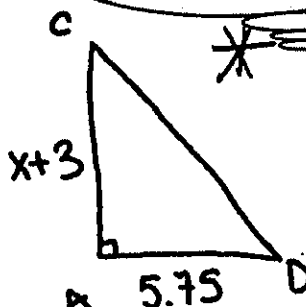
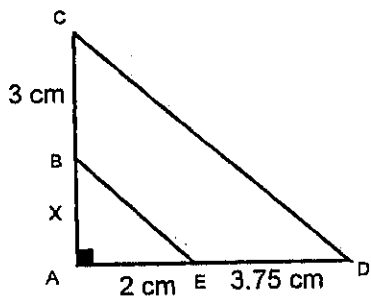
$$\frac{4x}{4} = \frac{7.2 \cdot 6}{4}$$

$$x = 10.8 \text{ cm}$$

Same answer $6 \times 1.8 = 10.8 \text{ cm}$

* Trick: Redraw separately *

b)



$$\begin{array}{ccc} \text{Big} & 5.75 & (x+3) \\ \text{Small} & 2 & x \end{array}$$

Distribute

$$2(x+3) = 5.75x$$

$$2x + 6 = 5.75x$$

$$2x - 5.75x = -6$$

$$-3.75x = -6$$

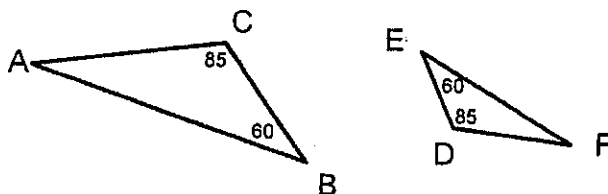
$$\frac{-3.75x}{-3.75} = \frac{-6}{-3.75}$$

$$x = 1.6 \text{ cm}$$

Minimal Condition for similar triangles

1. AA (Angle-Angle) Postulate

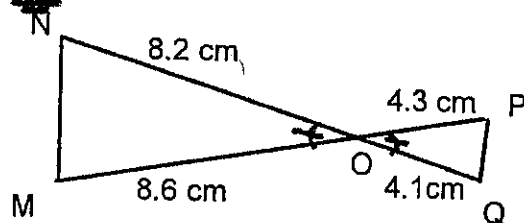
Two triangles are similar if they have 2 pairs of congruent **corresponding** angles.



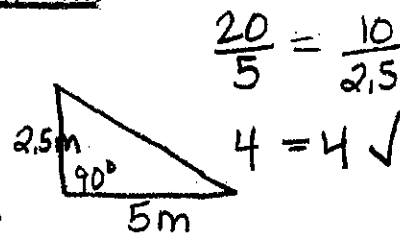
Use a postulate to state if two triangles are similar

2. SAS (Side-Angle-Side) Postulate

Two triangles are similar if they have 1 pair of congruent angles between 2 pairs of **proportional** sides.



OR



$$\frac{20}{5} = \frac{10}{2.5}$$

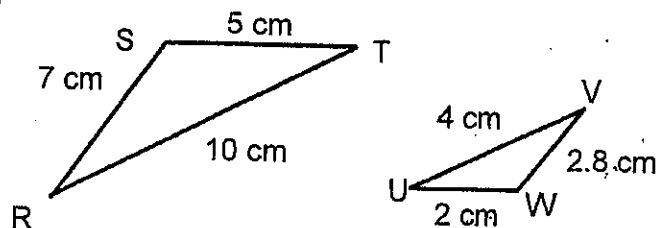
$$4 = 4 \checkmark$$

Check Sides: $\frac{8.6}{4.3} = \frac{8.2}{4.1}$ yes

(2) / (2)

3. SSS (Side-Side-Side) Postulate

Two triangles are similar if they have 3 **proportional** sides.



Check: $\frac{10}{4} = \frac{7}{2.8} = \frac{5}{2}$ \checkmark yes

2.5 2.5 2.5

